High-Level Outline

Firstly, I present certain positions in these notes as Tetronian's. Of course, they should be understood to be my understanding of Tetronian's position – they may or may not represent Tetronian's actual views.

At a high-level, the argument can be sketched as follows:

- Either idealism or materialism is true
  - There are other possibilities, such as dualism, but they are significantly less likely than either materialism or idealism, so we can get away with ignoring them
- Both materialism and idealism are equally compatible with all observations we have made heretofore of the universe
  - There are possible observations we might have, given which idealism is more likely than materialism. However, we haven't had any of these possible observations.
- Given the above, we are justified in ascribing to both materialism and idealism prior probabilities around 0.5 each.
- It is possible that our universe is a computer simulation. If our universe is simulated, it could be directly simulated in an unsimulated universe, or it could be part of a hierarchy of nested universe simulations, possibly a quite deep hierarchy – possibly even an infinitely deep hierarchy.
- For materialism to be true, whether our universe is simulated or not, there must be a basement universe. An infinitely deep simulation hierarchy, in which every universe is simulated in some other universe, with no basement unsimulated universe, is incompatible with materialism. Let us call a hierarchy incompatible with materialism a materialism-incompatible hierarchy.
- Whereas, idealism places no constraints on the simulation hierarchy of our universe.
- We don't know whether or not our universe has a materialism-incompatible simulation hierarchy. What is the probability that it has one? $P(incompat) = ?$
- So, $P(materialism) \cong P(idealism) \cong 0.5$ (materialism and idealism are the two main possibilities, and both are equally likely); $P(materialism|incompat) \cong 0$ (materialism is impossible, or close to impossible, if our universe has a certain sort of simulation hierarchy); $P(idealism|incompat) \cong 1$ (idealism is independent of the shape of the simulation hierarchy – it doesn't care)
- So it all comes down to the value of $P(incompat)$. Initially, we can say $P(materialism) = P(idealism) = 0.5$. If $P(incompat) = 0$, then we remain at $P(materialism) = P(idealism) = 0.5$. If $P(incompat) = 1$, then we end up with $P(materialism) = 0, P(idealism) = 1$. If $P(incompat)$ is between 0 or 1, we end up materialism less likely than idealism – although how much less likely depends on what value we choose for $P(incompat)$. If it is near zero, then materialism will only be slightly less likely than idealism. If it is around 50%, then we would end up with around 25% chance of materialism and 75% chance of idealism. If it is near 100%, we will end up with materialism's probability being close to 0 and idealisms close to 1.
- But how can we determine $P(incompat)$? If we have absolutely no information, we should choose a prior of 0.5, which would end up with idealism more likely than materialism.
- But maybe, by studying the possible structures of simulation hierarchies, we can develop a better prior for $P(incompat)$ than just 0.5? That is the question.

Structures of Simulation Hierarchies

Let us denote universe u is simulated in universe v, which is to say, universe v contains a (hyper)computer which simulates unverse u, as: $u \xrightarrow{sim} v$. Let us understand $u \xrightarrow{sim} v$ to refer to direct simulation only, which is to say, the (hyper)computer in universe v simulates universe u directly, rather than indirectly, such as by simulating some other universe w which in turn contains a (hyper)computer which simulates universe u. So in that later scenario, rather than saying $u \xrightarrow{sim} v$, we shall say $u \xrightarrow{sim} w$ and $w \xrightarrow{sim} v$. Of course, if we consider the transitive closure of $\xrightarrow{sim}$, which is $\xrightarrow{sim^*}$, we can then say that $u \xrightarrow{sim^*} v$.

Now, we can define the simulation hierarchy of a universe u as follows:

$$H(u) \stackrel{\text{def}}{=} \left\{ v \middle| u \xrightarrow{sim^*} v \right\}$$

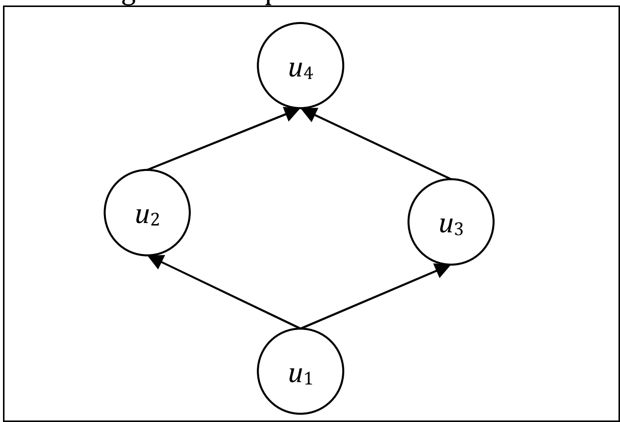
Self-simulating universes

An interesting question is, whether it is meaningful for a universe to be directly or indirectly self-simulating, i.e. $u \xrightarrow{sim} u$ or $u \xrightarrow{sim^*} u$. Rather than trying to answer this now, and to simplify discussion, let us just assume that this is impossible, hence $\forall u \cdot u \notin H(u)$. Since $u \notin H(u)$, let us define $H'^{(u)} \stackrel{\text{def}}{=} H(u) \cup \{u\}$.

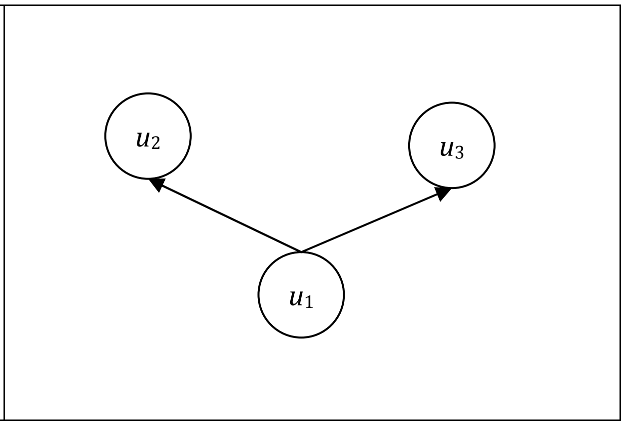
Originally, I tried to argue that if such circularity in the simulation hierarchy of our universe occurred, that would imply materialism as false. But, Tetronian disagrees with me, on the basis that a self-simulating universe is not really self-simulating, but rather one universe simulating a different universe, save that the two universes are identical. I address this position in more detail in the next section. In the meantime, let us just leave out circular hierarchies, and assume they are impossible.

Non-linear hierarchies

Note that it is, at least in principle, possible for hierarchies to be branching, such as in the following two examples:



Hierarchy A



Hierarchy B

It is an important question whether hierarchies could actually be non-linear as in either or both of the above examples. To help consider this question, some definitions would be useful:

Let us define that a universe u is hierarchy-linear if every universe that is simulated is directly simulated in only one other universe. Note this is upward-linearity, not downward-linearity – i.e. for two universes a and b to both be simulated in universe c is rather unproblematic, but for one universe a to be simulated directly in two separate universes b and c is much more peculiar.

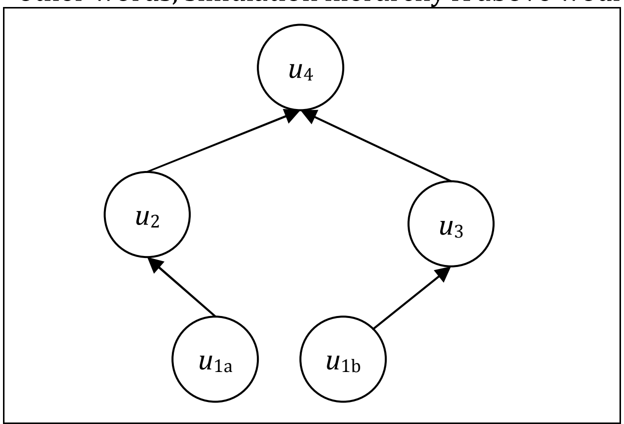
$$\text{linear}(u) \stackrel{\text{def}}{=} \forall a \in H'(u) \cdot \nexists b, c \in H'(u) \cdot a \xrightarrow{sim} b \land a \xrightarrow{sim} c.$$

In hierarchy A, $u_1 \xrightarrow{sim} u_2$, $u_1 \xrightarrow{sim} u_3$, $u_2 \xrightarrow{sim} u_4$, $u_3 \xrightarrow{sim} u_4$. Whereas, in hierarchy B, $u_1 \xrightarrow{sim} u_2$, $u_1 \xrightarrow{sim} u_3$. Arguably, there is something more “peculiar” about hierarchy B than hierarchy A. They are both non-linear, but B is fundamentally non-linear (multiple basement universes), whereas A isn’t (a single basement universe). B’s non-linearity is deep, A’s non-linerarity is shallow. How can we capture this “peculiarity” more formally?

$$\text{deeplinear}(u) \stackrel{\text{def}}{=} \forall a \in H'(u) \cdot \nexists b, c \in H'(u) \cdot a \xrightarrow{sim} b \land a \xrightarrow{sim} c \land H(b) \cap H(c) = \emptyset.$$

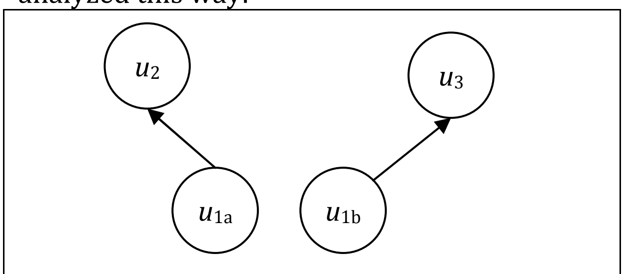
Informally, a universe is hierarchy-deeplinear, if it does not contain disjoint subhierarchies. So we can divided hierarchies into the linear, the shallowly non-linear (the non-linear yet deeplinear), and the deeply non-linear (the non-deeplinear).

Tetronian has made the argument that non-linear hierarchies can be reanalyzed as two separate hierarchies, containing two universes which are identical but-for their simulation hierarchy. In other words, simulation hierarchy A above would be re-analyzed as:



Hierarchy A’

Where universes u_{1a} and u_{1b} are two distinct but identical universes; i.e. identical in all their internal content, but differing in their simulation hierarchies. Likewise hierarchy B would be re-analyzed this way:



Hierarchy B’

Where likewise universes u_{1a} and u_{1b} are two distinct but identical universes; i.e. identical in all their internal content, but differing in their simulation hierarchies.

If we take Tetronian’s approach, then all simulation hierarchies would be linear. Alternatively, one might take an intermediate position – shallowly non-linear hierarchies are possible, but deeply non-linear ones are not.

I am unsure if this approach is correct. The principle of the identity of indiscernibles implies, that if two things are identical in all their properties, then they are in actuality the same thing. Clearly u_{1a} and u_{1b} are identical in all their *internal* properties; they differ only in their external ones. The question is, do the internal properties of a universe alone constitute its identity, or can its identity be constituted by its external properties (its relations to other universes) also.

Suppose our universe is one u_{1a} or u_{1b}. If we took Tetronian's approach, and hence $u_{1a} \neq u_{1b}$, then it would follow that there exists another person who is exactly alike me in every which way – with precisely the same thoughts and experiences as I have, at every moment of our respective existences – the only difference between us, would be I would exist in one of u_{1a} or u_{1b}, and the other person would exist in the other. I don't think that is possible – such a person would not be another person who is a precise replica of myself, that person would simply be myself.

However, we don't really have to resolve this issue here. If Tetronian's approach is correct, then all simulation hierarchies are linear; whereas if mine are correct, some may be non-linear, shallowly or even deeply. But, that will not change the outcome of my argument (in its current form); if anything, his approach makes things simpler, since we can exclude consideration of all the non-linear cases.

In an earlier form of my argument, I sought to argue that the possibility of our universe's simulation hierarchy being non-linear reduced the probability of materialism with respect to idealism. I think, if we take my approach to the identity of universes, my argument may still succeed; however, if we take Tetronian's approach, it clearly doesn't. So, I have decided to abandon that line of argument, at least for the time being.

Infinity and basementing

Note that every universe in a given hierarchy itself has a hierarchy:

$$\forall v \in H(u) \cdot\ H(v) \subset H(u)$$

Let us consider some properties of universes in terms of their simulation hierarchies. Firstly, a universe is unsimulated if $H(u) = \emptyset$ or equivalently $|H(u)| = 0$. So let us define:

$$\text{unsim}(u) \stackrel{\text{def}}{=} |H(u)| = 0$$

A universe is hierarchy-infinite if $|H(u)| = \infty$.

A universe is basementless if:

$$\text{basementless}(u) \stackrel{\text{def}}{=} \forall v \in H(u) \cdot |H(v)| = \infty$$

In other words, a universe u is basementless if every universe in its hierarchy is in turn simulated; in other words, there are no unsimulated universes in its hierarchy.

Conversely, a universe is basemented if:

$$\text{basemented}(u) \stackrel{\text{def}}{=} \exists v \in H(u) \cdot |H(v)| = 0$$

In other words, a universe is basemented if their exists at least one universe in its hierarchy which is unsimulated.

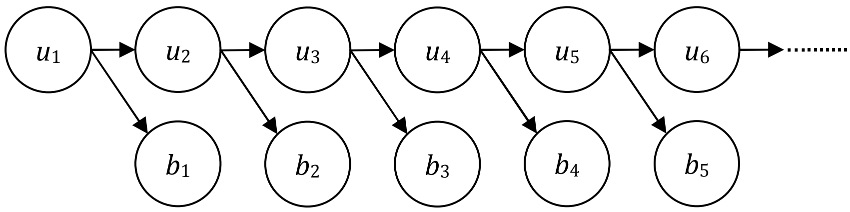
We can define the basements of a universe u, $B(u)$, to be the set of all basement universes of that universe, if any. If the universe is unsimulated, then it is its own basement universe. Thus, we will define $B(u)$ in terms of $H'(u)$ rather than $H(u)$ directly:

$$B(u) \stackrel{\text{def}}{=} \{v \in H'(u) | H(v) = \emptyset\}$$

In other words, a universe is basemented if $|B(u)| > 0$ and basementless if $|B(u)| = 0$. A universe u is semi-basemented if $\exists a, b \in H(u) \cdot |B(a)| = 0 \wedge |B(b)| > 0$. In other words, its hierarchy is composed of multiple branches, some of which end in basement universes, others constituting an infinite regress of universes. So every universe must be either basementless or basemented, and basemented universes can be either fully basemented (containing no infinite simulation regresses) or semi-basemented.

Clearly, $|B(u)| = 0$ implies $|H(u)| = \infty$.

Now consider the following hierarchy:



In this structure, there are an infinite number of basement universes, combined with an infinite regress of simulations. This structure is basemented, in fact it is fully-basemented. There is no universe v in the $H(u_1)$ such that $|B(v)| = 0$. Yet it nonetheless contains an infinite regress: $|H(u_1)| = \infty$.

Basementless and materialism

This argument relies on the following principles:

> **weak basementing principle**: if materialism is true, then our universe is not basementless.
>
> **strong basementing principle**: if materialism is true, then no universe is basementless.

If weak basementing is true, then our universe has a non-empty set of basements.

If materialism and strong basementing are true, then our universe must be fully basemented. This can be shown as follows: Suppose our universe was instead semi-basemented. Then there would be some universe in the simulation hierarchy of our universe which would be basementless – which by the strong basementing principle would be incompatible with materialism.

If linearity holds, as I think Tetronian argues – that is to say, all simulation hiearchies are linear, since (the argument goes) any non-linear hierarchy can be decomposed into two or more linear hierarchies, with some distinct yet internally identical universes, i.e. universes which are identical as to their internal properties but which differ in their relations to other universes – then for our universe to fully basemented, it must have a single basement, and its hierarchy must be finite.

I believe both the weak and strong basementing principles are intuitively true.

Probability that our universe is basementless

So, it all comes down to – what is the probability that our universe is basementless?

How could we give a better answer than 0.5? One approach seems to be as follows – assuming all possible universes are equally likely, what proportion of such universes are basementless? Or, if some possible universes are more likely than others, what is the relative likelihood of basementless compared to basemented universes?

Firstly, we must ask again, what is a universe? Or, more precisely, how can we construct models of universes?

One answer (I think Tetronian's) sees universe modelled by bitstrings – essentially, the perfect model of a universe is the shortest program which outputs the universe. I understand the implication to be that the bitstrings must be finite – which implies that, while the universe is not necessarily finite, it must have finite Kolmogorov complexity – and if it is infinite, it must be

countably so. Alternatively, if we permitted infinite bitstrings, we could have universes of infinite Kolmogorov complexity also, and also uncountably infinite universes.

Another, more physics-oriented answer, might proceed as follows: most physical theories assume the universe constitutes a *n*-dimensional spacetime, where *n* is most commonly 4, but in some theories larger – e.g. in string theory, *n* is either 11 or 27. This then implies the universe is a set of objects in a spacetime of n, where *n* is either 11 or 27; therefore, the universe is a set of objects in a spacetime of $\mathbb{R}^n$. Theories differ as to precisely what these objects are – waves, point particles, fields, strings, branes, etc. However, let us suppose that, whatever the fundamental geometrical objects of reality are, they are all decomposable into points. This decomposition may not have any physical meaning – e.g. if some version of string theory or M-theory is true, then strings or branes would be the smallest physically meaningful entities, and their decomposition into points would not be physically meaningful – the individual geometrical points that make up a string or brane cannot exist independently of the string or brane they compose. But, that such a decomposition is physically meaningless, does not mean it is mathematically/geometrically meaningless.

Another issue is that some physical theories in $\mathbb{R}^n$ are purely geometrical – the universe can be completely described by geometric objects in $\mathbb{R}^n$ – whereas others are not purely geometrical. Consider for example a simple theory in which point particles exist in an $\mathbb{R}^4$ spacetime, and these point particles have mass. This is not a purely geometrical theory, since these particles are not fully described by their spacetime position. But, I believe, every such theory can be transformed into an equivalent purely geometrical theory in $\mathbb{R}^n$, with some additional dimensions beyond the standard spacetime dimensions. This is exactly what general relativity, Kaluza-Klein theory, string theory and M-theory do. The precise details of such a transformation isn't relevant for our purposes, just that such a transformation exists.

The conclusion I am trying to reach is that given any reasonable physical theory, a universe can be expressed as a set of points in $\mathbb{R}^n$ for some finite integral *n*. Hence, a universe will be an element of $2^{\mathbb{R}^n}$ (that is, the powerset of $\mathbb{R}^n$). Since $|\mathbb{R}^n| = |\mathbb{R}|$ for all finite integral *n*, thus $\left|2^{\mathbb{R}^n}\right| = \left|2^{\mathbb{R}}\right|$. Now, we know that $|\mathbb{R}| = \left|2^{\mathbb{N}}\right| = 2^{|\mathbb{N}|} = 2^{|\mathbb{N}|} = 2^{\aleph_0} = 2^{\beth_0} = \beth_1$. (If we were to assume the generalised continuum hypothesis, or just the continuum hypothesis, we could also conclude $\beth_1 = \aleph_1$– but no need to bother with that.) Furthermore $\left|2^{\mathbb{R}^n}\right| = \left|2^{\mathbb{R}}\right| = 2^{|\mathbb{R}|} = 2^{\beth_1} = \beth_2$. So we can conclude that universe can be as large as $\beth_1$ and that there are as many as $\beth_2$ possible universes.

Now, if we insisted that universes were finite, then there would be only $\beth_1$ rather than $\beth_2$ possible universes. Furthermore, if instead of defining them over the reals, $\mathbb{R}$, we defined them only over the computable reals, we could pare them back even to $\beth_0$ for finite universes and $\beth_1$ for infinite universes. (Actually, infinite universes with finite Kolmogorov complexity would also number $\beth_0$.)

I don't we should not assume that all possible universe*s* will have the same dimensionality *n* – different universes may have different laws of physics. If this is a simulated universe, then the observed laws of physics in this universe may be radically different from those of the substrate universe. So, rather than being elements of $2^{\mathbb{R}^n}$, universes may be elements of $2^{\mathbb{R}^*}$, where $s^* = \bigcup_{n=1}^{\infty} s^n$. (Or maybe it is more accurate to say, elements of $\bigcup_{n=1}^{\infty} 2^{\mathbb{R}^n}$). I think the end result of this, is that $\left|2^{\mathbb{R}^n}\right| = \left|2^{\mathbb{R}}\right| = 2^{|\mathbb{R}|} = 2^{\beth_1} = \beth_2$, but $\left|2^{\mathbb{R}^*}\right| = \beth_3$. (I'm a bit unsure here though.)

Now, $\xrightarrow{sim}$ is a binary function $2^{\mathbb{R}} \times 2^{\mathbb{R}} \to \{0,1\}$. So, clearly, $\left|\xrightarrow{sim}\right| = \left|2^{\mathbb{R}} \times 2^{\mathbb{R}} \to \{0,1\}\right| = \left|2^{\mathbb{R}}\right| \times \left|2^{\mathbb{R}}\right| \times |\{0,1\}| = \left|2^{\mathbb{R}}\right| \times \left|2^{\mathbb{R}}\right| \times 2 = \left|2^{\mathbb{R}}\right| \times 2 = \left|2^{\mathbb{R}}\right| = \beth_2$. Alternatively, if we defined universes over $2^{\mathbb{R}^*}$, it may be that $\left|\xrightarrow{sim}\right| = \beth_3$.

Anyway, we really know almost nothing about the actual shape of $\xrightarrow{sim}$. So, ultimately, we can make no conclusion over how many possible universes are botomless, and hence what is the probability that our universe is bottomless.

I can only see three choices – refuse to pick a prior (we have no idea what one should be); pick one arbitrarily (well, whatever you want then really); just default to 0.5.

Some final diversions…

An anthropic aside: there are $\beth_0$ possible computable universes and at least $\beth_1$ possiible uncomputable universes. Assuming that both types of universes are equally likely to contain observers (which seems a valid assumption), and the average number of observers does not differ between universes (again, a seemingly valid assumption), there will be infinitely more observers in uncomputable than computable universes ($\beth_1 \gg \beth_0$). We don't have any clear evidence at the moment as to whether our universe is computable or uncomputable. In the absence of further evidence, we should conclude that almost certainly our universe is uncomputable.

By similar logic, since a finite powerset has the same cardinality as the base set, but the powerset is a beth number larger, we should conclude that in the absence of contrary evidence our universe is almost certainly infinite. (Current cosmological data is insufficient to come to a conclusion regarding the global topology of spacetime.)